2.3 Using Squares to Find Lengths

Goal

- Use geometric understanding of square roots to find lengths of line segments on a dot grid

In this problem, students develop a strategy for finding the distance between dots on a grid by examining the line segment between the dots. To find the length of the line segment, students draw a square with the segment as one side, find the area of the square, and then find the square root of the area.

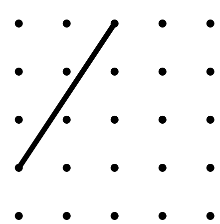
Launch 2.3

As a class, list all the side lengths (in units) students have found so far in their work with 5 dot-by-5 dot grids: 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$, 3, $\sqrt{10}$, and 4.

Suggested Question Ask:

- *Can you draw a line segment on a 5 dot-by-5 dot grid with a length that is different from these?*

On Transparency 2.3A, draw the segment the class suggests, or draw one of your own. Here is an example:



Suggested Questions Ask:

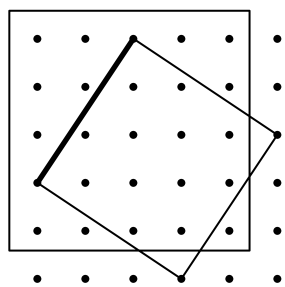
- *How do you know the length of this segment is different from others you have found?*

Students might mention ways to informally measure the length of the segment, or they might suggest comparing the segment to others that are a bit shorter or longer.

- *How might we find the actual length of this line segment?*

Some students might suggest drawing a square using this segment as a side and then calculating the segment's length from the square's area. If no one suggests this method, remind students of the connection they found between the area of a square and the length of a side. Walk through this process for the segment you drew.

Explain to students that the squares they draw will extend beyond the 5 dot-by-5 dot grid. For example, here is the square for the segment to the left below. The area of the square is 13 square units, so the length of the segment is $\sqrt{13}$ units.



When students understand the process, distribute Labsheet 2.3 and have students explore the problem in groups of three or four. If geoboards are available, students can put two or more of them together to work on this problem.

Explore 2.3

Groups do not need to find all 14 possible lengths. However, be sure every student is able to draw a square on a line segment and find the length of the segment. You may want to have some groups put their work on poster paper for discussion.

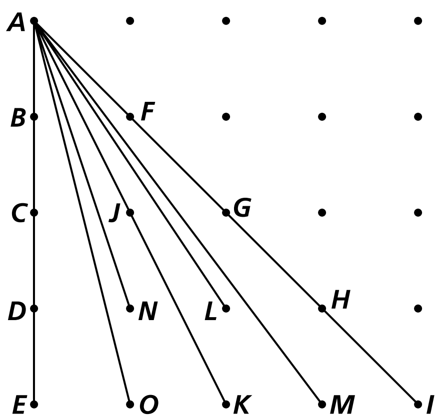
Going Further

Ask students who finish to count the different lengths that can be drawn on a 2 dot-by-2 dot grid, a 3 dot-by-3 dot grid, and a 4 dot-by-4 dot grid. Have them look for a pattern that will help them to predict the number of possible lengths on a 6 dot-by-6 dot and 7 dot-by-7 dot grid. For an n dot-by-n dot grid, there are all of the lengths that were in an $(n - 1)$ dot-by-$(n - 1)$ dot grid, plus n more. Therefore, a 6 dot-by-6 dot grid has the 14 lengths from the 5 dot-by-5 dot grid, plus 6 more, for a total of 20. The 7 dot-by-7 dot grid has $20 + 7$, or 27 lengths.

INVESTIGATION 2

Summarize 2.3

Ask students to share the lengths they found. Draw the lengths on Transparency 2.3A or show them on an overhead geoboard. Continue until all 14 lengths are displayed. Ask students to share strategies they used to make sure they had all the lengths. Arrange the lengths in an orderly way (see below).



Segment	Length (units)
AB	1
AC	2
AD	3
AE	4
AF	$\sqrt{2}$
AG	$\sqrt{8}$, or $2\sqrt{2}$
AH	$\sqrt{18}$, or $3\sqrt{2}$
AI	$\sqrt{32}$, or $4\sqrt{2}$
AJ	$\sqrt{5}$
AK	$\sqrt{20}$, or $2\sqrt{5}$
AL	$\sqrt{13}$
AM	5
AN	$\sqrt{10}$
AO	$\sqrt{17}$

Discuss the strategies students used to find the lengths. In some cases, students may have used relationships between line segments rather than drawing a square. For example, the length of segment AG is twice that of segment AF, so it is $2\sqrt{2}$. The area of a square with a side length of $2\sqrt{2}$ is 4 times the area of the similar square with an area of 2, or $4 \cdot 2 = 8$. Thus $\sqrt{8} = 2\sqrt{2}$. Students who find the length of AG by drawing a square will get $\sqrt{8}$. If your class is ready, talk about this equivalence: $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$. Or, have students use a calculator to evaluate the various expressions.

Suggested Questions To test their understanding of A(3), ask the following:

- *Between what two whole numbers does* $\sqrt{17}$ *lie?* (4 and 5)
- *Which whole number is it closer to?* (It is closer to 4 because 4.52 = 20.25, so $\sqrt{17}$ is less than 4.5. A calculator tells us that it is about 4.123105626.)
- *Between what two whole numbers does* $\sqrt{32}$ *lie?* (5 and 6)
- *How many of the lengths we have listed would you have found on a 4-dot-by-4-dot grid?* (1, 2, and 3 as side lengths of upright squares; $\sqrt{2}$, $\sqrt{5}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{13}$, and $\sqrt{18}$ as side lengths of tilted squares)
- *What is* $\sqrt{2} \times \sqrt{2}$? (2) *Why?* (Because $\sqrt{2}$ is the side length of a square with area 2)
- *What is* $\sqrt{5} \times \sqrt{5}$*?* (5) *Why?*

If Question C has not been discussed, be sure students share their strategies. Ask if there are other line segments whose lengths can be expressed in more than one way. For example, $3\sqrt{2} = \sqrt{18}$ and $2\sqrt{5} = \sqrt{20}$.

- *Are there lengths that cannot be expressed in more than one way?* (Yes, $\sqrt{2}$, $\sqrt{5}$ . . .)

Check for Understanding

Draw another segment on a dot grid. Ask the class to express its exact length using a $\sqrt{}$ symbol and then to tell which two whole numbers the length is between.

- *Which whole number is it closer to? How do you know?*
- *Is there another way to express this length? (For example,* $\sqrt{8} = 2\sqrt{2}$*)*

2.3 Using Squares to Find Lengths

At a Glance

PACING 1 day

Mathematical Goal

- Use geometric understanding of square roots to find lengths of line segments on a dot grid

Launch

Materials
- Transparency 2.3A
- Labsheet 2.3
- Centimeter rulers
- Geoboards (optional)

List all the side lengths that students have found so far in their work with 5 dot-by-5 dot grids: 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$, 3, $\sqrt{10}$, and 4.

- *Can you draw a line segment on a 5 dot-by-5 dot grid with a length that is different from these?*

On Transparency 2.3A, draw the segment the class suggests, or draw one of your own.

- *How do you know the length of this segment is different from others you have found? How might we find the actual length of this line segment?*

Explain to students that the squares they draw in the problem will extend beyond the 5 dot-by-5 dot grid. Have students explore the problem in groups of three or four.

Explore

Materials
- Transparencies 2.3B and 2.3C

Groups do not need to find all 14 possible lengths. However, be sure every student is able to draw a square on a line segment and find the length of the segment.

Summarize

Materials
- Student notebooks

Ask students to share the lengths they found. Draw the lengths on Transparency 2.3 or show them on an overhead geoboard. Continue until all 14 line segment lengths are displayed. Ask the class for strategies they used to make sure they had all the lengths.

Discuss the strategies that students used to find the lengths. If your class is ready, talk about equivalence: $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$.

Part (3) of Question A asks students for approximations of some of the square roots they have found. To test their understanding, ask the following:

- *Between what two whole numbers does $\sqrt{17}$ lie? Which whole number is it closer to?*
- *Between what two whole numbers does $\sqrt{32}$ lie?*
- *How many of the lengths we have listed would you have found on a 4 dot-by-4 dot grid? What is $\sqrt{2} \cdot \sqrt{2}$? What is $\sqrt{5} \cdot \sqrt{5}$? Why?*

If Question C has not been discussed, be sure students share their strategies. Share Transparencies 2.3B and 2.3C with your students.

continued on next page

Summarize *continued*

Check for Understanding

Draw another segment on a dot grid. Ask the class to express its exact length using a $\sqrt{\ }$ symbol and then to tell which two whole numbers the length is between.

ACE Assignment Guide for Problem 2.3

Core 35–37, 41

Other *Applications* 38–40; *Connections* 43–46; *Extensions* 49–53; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercise 41 and other ACE exercises, see the CMP *Special Needs Handbook.*

Connecting to Prior Units 43: *Covering and Surrounding;* 45: *Bits and Pieces III;* 46: *Stretching and Shrinking*

Answers to Problem 2.3

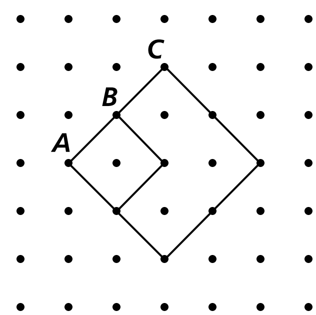
A. 1 and 2.

The possible lengths in increasing order are 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$, 3, $\sqrt{10}$, $\sqrt{13}$, 4, $\sqrt{17}$, $\sqrt{18}$, $\sqrt{20}$, 5, and $\sqrt{32}$. See the Summarize section for pictures and more information.

3.

Exact Length	Decimal Approximation
$\sqrt{2}$	1.4
$\sqrt{5}$	2.2
$\sqrt{8}$	2.8
$\sqrt{10}$	3.2
$\sqrt{13}$	3.6
$\sqrt{17}$	4.1
$\sqrt{18}$	4.2
$\sqrt{20}$	4.5
$\sqrt{32}$	5.7

B. Both are correct. The length of AC is twice the length of AB. Because the length of AB is $\sqrt{2}$ (being a side of the small square), the length of AC is 2 times $\sqrt{2}$, or $2\sqrt{2}$. We can also find the length of AC directly by making it a side of a square (the large square in the picture below) whose area is 8. So, the length of AC also equals $\sqrt{8}$. So, $2\sqrt{2} = \sqrt{8}$.



C. 1. $\sqrt{40}$, or $2\sqrt{10}$

2. Some examples are $\sqrt{17}$, $\sqrt{13}$, $\sqrt{5}$.